A tube conveying flow becomes unstable as the flow rate increases, and shows chaotic behavior. We investigate the 3D dynamics of a hanging tube conveying air flow with varying the length of the tube and the flow rate. Two different boundary conditions at the hanging end of the tube are applied; one is the rigid end, and the other is axially rotatable. We find instabilities to static buckling, plane or rotating pendulum, sub-harmonic or to chaotic states, depending on boundary conditions. The power spectra of the tube’s motion show characteristic peaks and the “exponential decay” law. We classify the tube’s motions according to their spectra and draw phase diagrams.

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I. INTRODUCTION

Linear structures containing flow inside them are found commonly in the wide range of length scale. The macroscopic example is a pipeline for the oil industry, and another is found inside our bodies, namely a blood vessel. Of all such things, the simplest and most popular one is a hose conveying water found in a garden or a scene of fire-fighting.

Intuitively, it seems that such a tube-like structure can be unstable and will rush about wildly for the powerful flow inside it, but in the context of physics it is not obvious. The dynamics of tube conveying flow has been a classical problem both in experimental and theoretical fluid dynamics [1–7]. It is well-known that Feynman also inquired into the problem of a sprinkler sucking in water [8]. Recently, Shima and Mizuguchi have constructed a 2D model of tube with flowing fluid inside it and investigated its instability both analytically and numerically [9,10]. However, to our knowledge, this theme is not sufficiently explored experimentally in view of the 3D dynamic properties of a tube discharging fluid without constraint on its free end. In this paper we study experimentally the 3D dynamics of hanging tube conveying fluid.

In the following, the setup of the present experiment and the method of the data analysis are described. Two boundary conditions are applied for the hanging end of the tube, and for each cases, for a fixed length of the hanging tube, we illustrate the typical observation when the speed of fluid flowing in the tube increases. After that, the length of the tube is varied and the phase diagrams as a function of the length of tube $l$ and the flow rate $Q$ are sketched. And finally the effects of the natural form of the tube on the bifurcation are discussed briefly.

II. METHODS

The experimental setup is shown in Fig.1. A silicon tube is connected to a thin brass tube of length 2cm then hanged by a pillow block on a support. The height of the support is 1.5m, that is enough to avoid the effect of back-flow ejected from the free tail of the tube and reflected by floor. We can apply different boundary conditions on the brass tube hanging the silicon one. The most natural boundary condition is seemed to be the “clamped” condition that fixes the hanging end rigidly and forbids any axial rotation. However, this condition is accompanied with the effect of torsion, and the effect becomes one of the dominant elements of the dynamics in that case. Therefore another boundary condition is also tested, namely, the “axially rotatable” condition that enables the tube to rotate around the symmetry axis of the system. For axially rotatable boundary condition, the brass tube is connected to a sealed ball bearing system.

FIG. 1. Experimental setup. A compressed air cylinder (a) provides the air to the tube (d) hanging from a pillow block (c), with measuring the flow rate of air by mass-flow meter (b). The movement of the tube is recorded by the digital camcorder (e) below.

The internal and external diameters of the tube are 2mm and 4mm, respectively. Our tube is a custom-made
article that has no natural distortion and has straight form in nature, whereas tubes on the market are wound and take curved forms even without any stress.

Pressure regulated air is led into the tube from the hanging end. The flow rate is measured at the entrance of tube by a mass-flow meter. In this experiment, the measurable range of the flow rate $Q$ is $Q \lesssim 50\ell/\text{min}$, with the tube’s length is on the order of 10cm.

A digital camcorder is placed directly below the tube and records the projection of the tube’s motion onto a horizontal plane. The sampling rate of the movie is set at 60Hz with a spatial resolution of 640x480 pixels. The fastest characteristic time scale of the tube’s motion is approximately 30Hz, therefore the rate provides sufficient resolution. The speed of electrical shutter of each image is 1/750 sec, otherwise stated.

The movies recorded by the camcorder are sent to PC and image processing are performed by using NIH Image software. To characterize distinct dynamic states, we measured orbits of a representative point on the tube and calculated the rotational power spectrum. For the representative point, we choose the center of mass coordinate $(\bar{x}(t), \bar{y}(t))$ on the horizontal projection plane defined as

$$\bar{x}(t) = \sum_i x_i(t)s_i(t) / \sum_i s_i(t), \quad \bar{y}(t) = \sum_i y_i(t)s_i(t) / \sum_i s_i(t),$$

where $(x_i(t), y_i(t))$ is the Cartesian coordinate of a segment $i$ of the tube and $s_i(t)$ is its area obtained by segmentation processing for each time slice $t$.

Due to the symmetry of our system, it is more convenient to define a complex coordinate

$$z(t) \equiv \bar{x}(t) + i\bar{y}(t)$$

and treat our problem as a dynamics of the single point $z(t)$ on the complex plane. The dynamics of $z(t)$ are analyzed by calculating Fourier spectrum. We compute FFT amplitudes

$$\zeta(\omega) \equiv \sum_t z(t)e^{i\omega t},$$

and define the rotational power spectrum of $z(t)$ as

$$I(\omega) \propto |\zeta(\omega)|^2.$$  

In view of $I(\omega)$, the motions of the tube are classified. Note that, in our method, $z(t)$ is the complex variable and the positive frequency modes $e^{i\omega t}$ and the negative ones $e^{-i\omega t}$ into which $z(t)$ is decomposed contribute independently. Their amplitudes $\zeta(\omega)$ and $\zeta(-\omega)$ serve distinct informations, in contrast to the Fourier analysis of real functions in which $\zeta(\omega)^* = \zeta(-\omega)$. The rotational power spectrum analysis therefore enables us to extract all informations available from full range of the sampling frequencies and is compatible with complex numbers.

III. RESULTS

A. In case of the “clamped” boundary condition

At first, we choose the “clamped top” boundary condition. Various modes are observed as shown in the figures that appear later. We describe the typical transitions of tube’s motion with fixing its length and increasing the flow rate of air.

![FIG. 2. The snapshot of the static buckling state. The shutter speed is set to 1/30 sec for better visibility by an afterimage. The hanging tube exhibits no curvature without any applied force, but leading slow flow into it, it comes to take a curled form as is seen in this figure.](image-url)
However, with increasing the flow rate of air, the curvature of buckling becomes greater and the static buckling state becomes unstable. The tube begins to oscillate spontaneously, and turns into dynamic modes that are stable with respect to any perturbations. The first mode is the “pendulum” mode. It is the state that the tube oscillates periodically like a pendulum. Note that the motion of the center of mass on a horizontal plane is the round trip on a linear trajectory (see Fig. 3). The power spectrum shows odd harmonics. It is reasonable, because the wave modes on the hanging tube have to satisfy the conditions that one end is fixed and the other is left free. We also find that the peak heights seen in the power spectra decay exponentially with respect to the frequency.

When the flow rate increases, the trajectory of the tube’s center of mass on the horizontal plane deforms into a figure of eight, and the next “buckled pendulum” mode appears. The trajectory is closed for one cycle around the origin as shown in Fig. 4. It is not the simple deformation of a straight-wired pendulum, that is understood by observing its power spectrum. In addition to the odd harmonics, the even harmonics peaks also contribute significantly. Note that the “exponential decay laws” for odd and even peaks have different decay rate and also are different in their absolute heights.

The figure-eight trajectory of the buckled pendulum mode gradually flutters with increasing the flow rate as shown in Fig. 5, and comes to the “sub-harmonic” mode. The fluttering is accompanied with torsion of the tube because of the clamped boundary condition, and this torsion sustains the fluttering. The center of mass rounds more than one cycle around the origin to close its orbit on the horizontal plane. Turning our eyes to the 3D motion of tube, it exhibits fluttering up and down, so in reality this is not a two dimensional movement. In the power spectrum of the mode, additional features are found; the sub-harmonic peaks. We can recognize one-third peaks, four-thirds peaks, and so on, of the basic frequency. These fractional harmonics are the analog of that found in Duffing equation caused by the cubic non-linearity. By our observation of their power spectra, it turns out that these sub-harmonic peaks appear after the “exponential decay” of the odd and even peaks come to
coincide. We also notify that stroboscopic plot of the trajectory and power spectrum for the sub-harmonic mode show some characters of weakly chaotic state as is shown in Fig. 5, especially for the higher flow rate.

FIG. 5. The stroboscopic plot (left) and the power spectrum (right) of the subharmonic mode at \( l = 10\text{cm} \) and \( Q = 43.0\ell/\text{min} \). In the power spectrum, We can clearly see one-third peaks, four-thirds peaks, and so on, of the basic frequency \( \sim 10\text{Hz} \).

The subharmonic peaks grow gradually together with rising all kind of rational subharmonics. Finally the tube arrives at the truly “chaotic” mode (see Fig. 6), and the gaps between the integer harmonic peaks are filled up densely. In the 3D physical space, the tube twists and bends itself, and occasionally strikes the top of the hanging frame or the very tube itself. The projection of the motion onto a 2D plane exhibits totally chaotic trajectories, all the more in 3D space. The only one law found in the power spectrum of the chaotic mode is that no characteristic peak appears and in average it decays exponentially with respect to the frequencies.

FIG. 6. The stroboscopic plot (left) and the power spectrum (right) of the rotating pendulum mode at \( l = 16\text{cm} \) and \( Q = 33.2\ell/\text{min} \). The trajectory is a nearly straight line which rotate slowly around the origin (only a part of an oscillation cycle is shown), and the slow rotation is reflected as a peak at \(-20\text{mHz}\) in the power spectrum.

Thus we can easily get the “rotating pendulum” mode as shown in Fig. 7. The tube oscillates like a plane pendulum, together with a slow rotation of the plane of oscillation around the axis of the system. This mode is periodic and the odd harmonic peaks that correspond to the pendulum-like oscillation appear in its power spectrum. The “exponential decay” is realized for them as before. The lower peak at about \(-20\text{mHz}\) also found in the spectrum corresponds to the clockwise rotation of the plane of the oscillation. This slow rotation of the plane of oscillation couples with the oscillation of pendulum. Mixing of the even harmonic peaks and the slow rotational mode produces frequency peaks of mixed modes as shown in Fig. 7.

B. In case of the “axially rotatable” condition

Axial twisting is one of the dominant elements of this system as mentioned above. Now this feature is removed by arranging the “axially rotatable top” boundary condition which cancels the axial distortion for dynamic modes.

At the beginning of leading air flow into the hanging tube, the buckling state appears as well, but now from the beginning this is an unstable state with respect to perturbations, and changes into rotating pendulum mode. This is because the axial twisting of the tube is removed by the free axial rotation and the removal enables it to recover the symmetries of the system which was lost by the buckling. The transition from buckling state to rotating pendulum mode is a subcritical bifurcation.

FIG. 7. The trajectory (left) and the power spectrum (right) of the rotating pendulum mode at \( l = 20\text{cm} \) and \( Q = 26.2\ell/\text{min} \). The trajectory is a nearly straight line which rotate slowly around the origin (only a part of an oscillation cycle is shown), and the slow rotation is reflected as a peak at \(-20\text{mHz}\) in the power spectrum.
FIG. 8. The trajectory (left) and the power spectrum (right) of the rotating buckled pendulum mode at \( l = 16\text{cm} \) and \( Q = 37.4\ell/\text{min} \). The trajectory takes ellipsoidal form but turning its major axis rather chaotically. As a result, the peaks seen in the power spectrum lose their sharpness and the generation of sharp sub-harmonic peaks is suppressed.

The plane of oscillation gradually deforms, and the trajectory of the tube’s center of mass on the horizontal plane also changes its shape from a rotating straight line to a rotating ellipse as seen in Fig.8. This “rotating buckled pendulum” mode is rather chaotic, because the rotation of the ellipse is not regular but suddenly the major axis of the ellipse revolves to a point to an unpredictably different direction. Therefore, strictly speaking this mode should be labeled with the phrase “chaotic,” but we save the phrase for the final mode that exhibits entirely unpredictable 3D motions. Returning to the subject, we note that the chaotic rotation of the major axis of the ellipse suppresses the generation of sharp sub-harmonic peaks in the power spectrum. For the sake of the suppression, the integer peaks lose its sharpness and spread extensively.

FIG. 9. The power spectrum of the “window” of chaos at \( l = 16\text{cm} \) and \( Q = 38.0\ell/\text{min} \). It shows the same nature which found in the rotating pendulum mode.

Before arriving at the final “chaotic” mode, the revival of the “rotating pendulum” is observed for some sets of values of parameters. As shown in Fig.9, the power spectrum recovers the sharpness of its peaks and clearly indicates the existence of order. This phenomenon is considered as the “window” of chaos.

Passing through the recurrence of the rotating buckled pendulum beyond the “window”, eventually the “chaotic” mode appears (see Fig.10). This mode has the same natures that are found in the “clamped” boundary condition.

FIG. 10. The stroboscopic plot (left) and the power spectrum (right) of the chaotic mode at \( l = 16\text{cm} \) and \( Q = 46.2\ell/\text{min} \). This mode exhibits the same nature seen for that of the “clamped” boundary condition.

C. 2D phase diagrams

In the description above, the length of the tube \( l \) is fixed and the flow rate \( Q \) is therefore the only parameter of measurements. The next thing to be studied is to observe with varying \( l \). Figure 11 is the 2D phase diagrams for each boundary condition. The transition points between different modes and their fitting curves are drawn.

In the diagram of the “clamped” condition, the pendulum mode and the buckled pendulum modes coalesce around \( l \approx 18\text{cm} \) and the bifurcation of tube’s motion is changed in the region of \( l \gtrsim 20\text{cm} \). In this region, as increasing the flow rate of air with a fixed length, the transition from the buckling state to the sub-harmonic mode is observed, and the chaotic mode appears with the increment in the quantity of air flow of \( \Delta Q \approx 5\ell/\text{min} \). For this boundary condition, there is a general trend that the critical values of the flow rate are nearly inversely proportional to the length of the tube.

On the other hand, the diagram of the “axially rotatable” boundary condition exhibits different nature. The transition lines in this condition are generally 10\ell/\text{min} higher than those of the “clamped” condition. Some “windows” of chaos that are not found in the “clamped” condition appear, and the most different feature is that the critical line of the transition from the rotating buckled pendulum mode to the chaotic mode falls slowly with respect to the length of the tube and keep its position in the region of \( Q > 40\ell/\text{min} \) within our exploration. Because of the elimination of the effect of torsion, the suppression of growing of the sharp sub-harmonic peaks prevents the tube from rushing into the chaotic mode in the wide range of the flow rate.

In both boundary conditions, determining the bifurcation points from a straight line to the buckled state is difficult within the accuracy of the present setup.
FIG. 11. The phase diagrams for the “clamped” condition (top) and the “axially rotatable” condition (Bottom). In the top diagram, circles, boxes, diamonds, and crosses represent the points of transition from the buckling (B) to the plane pendulum (P), from the plane pendulum to the buckled pendulum (BP), from the buckled pendulum to the subharmonics (SH), and from the subharmonics to the chaos (C), respectively. In the bottom diagram, circles, boxes, diamonds represent the transition from the buckling (B) to the rotating pendulum (RP), from the rotating pendulum to the rotating buckled pendulum (RBP), from the rotating buckled pendulum to the chaotic (C) state, respectively. In addition, the “windows” (W) of chaos (crosses) are observed.

D. The effect of natural form of the tube

Our tube used above has no natural distortion, but tubes on the market are by nature in curved form. Using such a naturally curled tube brings entirely different results. The curvature of the tube breaks the axial symmetry that was inherent in our hanging tube system, and induces a rotational state in which the tube swings with making a conic locus under the “axially rotatable” boundary condition. In addition, for both boundary conditions, irregular or periodic but strange-formed movements are also induced accidentally. For example, Fig. 12 shows a stable rotating 1-node mode observed rarely under the “axially rotatable” boundary condition. We note that Fig. 12 displays the surface of revolution of the vase-like structure formed by revolving 1-node mode. The hanging tube rotates around the symmetrical axis of the system keeping a curved 1-node shape, and if the flow rate of flowing fluid is increased this state doesn’t collapse into other modes but just enhances curvature until the flow rate arrives at the transition point of chaos.

FIG. 12. The snapshot of a stable rotating 1-node mode at \( l = 30 \text{cm} \) and \( Q = 16.3 \text{ℓ/min} \) observed rarely under the “axially rotatable” boundary condition. The shutter speed is set to 1/8 sec for better visibility by an afterimage. The hanging tube rotates around the symmetry axis of the system keeping a curved 1-node shape. This state doesn’t collapse into other modes increasing the flow rate of flowing fluid, until the flow rate arrives at the transition point of chaos.

The most striking feature brought by the curving na-
ture is that the transition points measured by varying the flow rate at fixed tube length show hysteretic behavior. In fact, when the flow rate is reduced the transition occurs at lower flow rate. Within our exploration using a curled tube the hysteresis is observed for both “clamped” and “axially roratable” boundary conditions, whereas only for the latter condition the hysteretic behavior is seen with a tube having no natural curvature. In order to describe the effect of the tube’s natural form and subsequent symmetry breaking quantitatively, the geometry and dynamics of tube in truly 3D space must be treated, and that needs both the improvement of experimental technique and more machine power for the spectral analysis.

IV. DISCUSSION

Since early times, many theoretical physicists and mathematicians have tried to describe the motion of tube conveying fluid and to clarified its stability from the perspective of mechanics [4]. The most intuitive derivation of the equation of motion is seen in the work of Gregory and Paidoussis [2] considering a tube discharging incompressible fluid on a horizontal x-y plane with clipping one end at the origin and lying along the x axis initially. The derived equation is as follows:

$$EI \frac{\partial^4 y}{\partial x^4} + M \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 y + m \frac{\partial^2 y}{\partial t^2} = 0,$$  
(5)

where $E$, $I$, $M$, $m$, $U$, $y$ are Young’s modulus of tube, inertial moment with respect to the axis perpendicular to the axis of tube, mass of tube per unit length, a mean flow velocity of fluid, transverse deflexion of tube measured perpendicular to the $x$ axis, respectively. Equation 5 has a quite reasonable structure from the view point of Newtonian dynamics; i.e. the tube’s motion (the third term in the left hand side of Eq.5) is determined by the elastic force (the first term in the L.H.S. of Eq.5) and the impulsive applied by the fluid (the second term in the L.H.S of Eq.5). However, this formalism is valid only for an infinitesimally small $|y|$.

Recently, Shima and Mizuguchi have formulated 2D dynamics of the tube conveying fluid, and their formalism is applicable not only for small deformation of tube but also for the motion of large amplitude. [9,10]. For reference to readers, the equation of motion describing the dynamics of a straight-natured tube discharging fluid on the 2D $x$-$y$ plane according to Reference [10], with a correction on it from the viewpoint of fluid dynamics and with the gravitational acceleration $g$ in $x$ direction, is written down as follows:

$$-\int_s^L ds' \int_0^{s'} ds'' \cos \{ \theta(s'', t) - \theta(s, t) \}$$

which is written down as follows:

$$\{ (\rho + \sigma_w) \frac{\partial^2}{\partial t^2} + \gamma \frac{\partial}{\partial t} \} \theta(s'', t)$$

\( + (\rho + \sigma_w) \int_s^L ds' \int_0^{s'} ds'' \left( \frac{\partial}{\partial t} \theta(s'', t) \right)^2 \times \sin \{ \theta(s'', t) - \theta(s, t) \} \)

\(- 2\sigma_w v_w \int_s^L ds' \left( \frac{\partial}{\partial t} \theta(s', t) \right) \times \cos \{ \theta(s', t) - \theta(s, t) \} \)

\( + \sigma_w v_w^2 \sin \{ \theta(s, t) - \theta(1, t) \} + \alpha \frac{\partial^2}{\partial s^2} \theta(s, t) \)

\(- (\rho + \sigma_w) g(L - s) \sin \theta(s, t) = 0, \)

(6)

where $t$ is time, $s \in [0, L]$ is the arc length of tube measured along it from its clipped top at the origin of the Cartesian coordinates, $\rho, \sigma_w$ are the densities per unit length of tube and fluid flowing inside it, respectively. $\alpha(\geq 0), \gamma(\geq 0), v_w$ represent the stiffness of the tube, the the strength of the resistance which is caused by the frictional interaction with a surrounding medium, the flow rate of the fluid, respectively. The dynamics of tube is described as a dynamics of the angular variable “field” $\theta(s, t)$ which represents the angle between the tangential vector of tube at $(s, t)$ and the $x$ axis. Equation (6) contains integral terms because of the restriction that the length of the tube $L$ is fixed as a given value.

In general, there are difficulties in constructing the 3D version of the dynamic model of tube conveying fluid, especially in choosing some “proper fields” and coordinates that don’t make the equations of motion singular. Thus there find no reasonable 3D theoretical model to be compared directly to our experimental study quantitatively, and we compare the present experimental results with existing theories regardless of the difference in their dimensionalities.

In Reference [9,10], Shima and Mizuguchi reduced their original equation of motion into Kuramoto-Sivashinsky type equation with peculiar boundary conditions in some physical limits, and found transition from a stationary straight state to an oscillating state for the clamped boundary condition, and to a rotating state for the rotatable boundary condition. Both transitions were found to be supercritical bifurcation. They did not observe a stationary buckled state for an idealized tube having straight form in nature. On the other hand, Lundgren et al. analyzed the case when the axial symmetry is slightly broken in the absence of gravity [3]. They showed that the first bifurcation from the straight state is the stationary buckled state when the free end of the tube is curved. In contrast with Lundgren et al., Benjamin [1] pointed out the fact that for an ideally straight-natured tube buckling couldn’t occur without gravity. In reality, the density $\rho$ of the tube used in our experiment is on the order of $10^3 \text{kg/m}^3$ and gravity is rather dominant in the regime of low flow rate. We cannot find any rea-
son for a justification of the neglection of gravity done in the theory of Lundgren, but we also cannot say which analysis gives correct explanation for the buckling seen in our experiment. To make a decision for this issue, the horizontal analog of our experiment in which gravity is insignificant should be performed, and in that case some technique which enables the tube laid on the horizontal plane to move without friction will be needed.

Apart from the issue discussed above, symmetrical aspect is quite important for our experimental setup. For the custom made tube which has almost straight form in nature buckling occurs as a spontaneous symmetry breaking, following the bifurcations into dynamical modes having periodicity and symmetry in their trajectories. In addition, the tube on the market which has curvature in nature exhibits various geometrical motions that maybe reflect the initial degree of symmetry breaking of the system. Classification in the perspective of symmetry can provide another informative method in analyzing this tube system, as seen in Reference [7].

Let us return to the question raised by Feynman: whether the sprinkler sucking in water can rotate or not? In Reference [8], he mentioned on the problem and said to have reached his own conclusion, but the conclusion was not described clearly. Even today it is really difficult to demonstrate the water-sucking sprinkler in Feynman’s setup, and many people discuss on the issue theoretically. For instance, there is no symmetry in Equation (6) with respect to the simple reversal of flow rate $v_w \leftrightarrow -v_w$, therefore asymmetric behavior between the tube discharging fluid and the one aspirating is expected if Equation (6) is correct. In early works such as Reference [6] the asymmetry with respect to the reversal of the flow is indeed seen both theoretically and experimentally, though only the free end of the tube was put in the water.

We can also consider the problem from the viewpoint of the symmetry of Navier-Stokes equation. Time reversal symmetry is approximately held only at a vanishingly small Reynolds number. At a moderate flow rate, ejected flow from the free end of the tube or sprinkler creates a jet, in which both inertial and viscous effects are not negligible. Flow pattern is not symmetric for the reversal of the flow in such a situation, nor the motion of the tube. We also notify that the Reynolds number is $Re \sim 10^4$ inside the tube, and $Re > 10^5$ in the jet at the highest flow rate in the present experiment. These high values of $Re$ number are enough to create turbulence.

The turbulent effect of jet at the outlet of tube somewhat appears in the discrepancy between the nature of dynamics of tube and that in a simple elastic pendulum. In Section III, characteristic harmonic peaks are found from the power spectra of dynamic modes of the tube. Typically, the fundamental frequency peak is found at a natural frequency $f_0$ and others at multiples of $f_0$. As described in Reference [2], the fundamental frequency $F_0$ of tube’s vibration as a simple elastic pendulum under the gravitational field is determined according to the formula

$$\frac{(2\pi F_0)^2 l}{g} = \frac{81}{52} + \frac{162 EI}{13 mgl^3}, \quad (7)$$

where $g, E, I, l, m$ are the gravitational acceleration, Young’s modulus, inertial moment with respect to the axis perpendicular to the axis of tube, length of tube, mass of tube per unit length, respectively. $E$ of silicon rubber tube can vary depending on sulfurazing reaction in manufacture, and the measured value of $E$ for our custom made tube is 3.1MPa. According to Eq.(7), we obtain $F_0 = 3.9Hz$ for $l = 10.0cm$, whereas $f_0 \approx 10Hz$ in our observation as seen in Figure 3. This marked difference of the basic frequency between the observed $f_0$ and the elastic-theoretical $F_0$ implies that dynamic modes are dominated by the reaction from the flow inside and outside of the tube, neither by elasticity nor by gravity. For rigorous analysis, nonlinearly coupled equations of motion for tube and fluid are needed, that is a hard problem of theoretical fluid dynamics.

V. CONCLUSION

The hanging tube being straight in nature exhibits dynamic motions induced by the air flow led into it. From the perspective of the power spectrum of its motion, the dynamical modes can be classified for each boundary conditions. The torsion of the tube caused by its motion and the restriction of its motion of the hanging top is one of the relevant characters and the removal of this realized by the axially rotatable boundary retards the appearance of subharmonic peaks and the chaotic mode. The system is also sensitive to the symmetry breaking brought by the natural curvature of tube, that is seen as the drastic changes of the phase diagram and the hysteretic behavior observed by varying the flow rate at fixed tube length. The detailed measurement of the phenomenon is a subject of interest remained for future studies. For more qualitative and quantitative understanding of the dynamic behavior of tube conveying fluid, the well-defined theoretical model in 3D should be constructed, in which the tube and the turbulent flow flowing inside it are maybe coupled nonlinearly.

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